

SU(5) \times SU(5) unification revisited

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ABSTRACT: The idea of grand unification in a minimal supersymmetric SU(5) \times SU(5) framework is revisited. It is shown that the unification of gauge couplings into a unique coupling constant can be achieved at a high-energy scale compatible with proton decay constraints. This requires the addition of a minimal particle content at intermediate energy scales. In particular, the introduction of the SU(2)_L triplets belonging to the (15, 1) + ($\overline{15}$, 1) representations, as well as of the scalar triplet Σ_3 and octet Σ_8 in the (24, 1) representation, turns out to be crucial for unification. The masses of these intermediate particles can vary over a wide range, and even lie in the TeV region. In contrast, the exotic vector-like fermions must be heavy enough and have masses above 10^{10} GeV. We also show that, if the SU(5) \times SU(5) theory is embedded into a heterotic string scenario, it is not possible to achieve gauge coupling unification with gravity at the perturbative string scale.

KEYWORDS: Grand unified theories; SU(5) \times SU(5)

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1 Introduction

On the quest for the theory beyond the Standard Model (SM), supersymmetric grand unified theories (SUSY GUTs) have revealed many attractive features which can solve some of the aspects left unexplained in the SM. This idea is supported by the unification of the gauge couplings that occurs, through renormalization group evolution, at a scale around 10^{16} GeV in the minimal supersymmetric standard model (MSSM). In the latter case, the SUSY threshold is set in the TeV region.

Since the appearance of the simplest GUT models proposed in 1974 by Georgi and Glashow, and based in the gauge group $SU(5)$ [1], the search for gauge groups compatible with a unification scheme has been actively pursued in the literature [2–4]. Yet the unification and breaking patterns are far from being established. The low-energy supersymmetric $SU(5)$ version [5] has been quoted as an excellent unification theory, since in this model gauge couplings unify very precisely at one-loop level without the need of new particles. Moreover, at two-loop [6] and three-loop [7] levels, gauge unification can also be achieved if threshold effects are taken into account.

Besides being successful in unifying gauge couplings, GUTs should also address other theoretical challenges. The proton should live long enough [3, 8–11]. This requirement usually leads to the well-known doublet-triplet splitting problem, i.e. the $SU(2)_L$ doublet and the $SU(3)_C$ colour triplet belonging to the same multiplet must have a strong mass hierarchy. In other words, the parameters in the Higgs potential responsible for the doublet and triplet masses must be highly fine tuned.

Going beyond the simplest $SU(5)$ unification, it is also conceivable that the unification group has a semi-simple structure, as in the original left-right symmetric

Pati-Salam model [12, 13]. In this direction, the SUSY left-right $\text{SU}(5) \times \text{SU}(5)$ model [14, 15] has many attractive features that are absent in minimal realizations of the $\text{SU}(5)$ theory. Indeed, R-parity can be automatically conserved, proton decay is suppressed because heavy and light fermions do not mix, the doublet-triplet splitting problem is alleviated [15, 16], a generalized seesaw mechanism for fermion masses can be easily incorporated, and nonvanishing neutrino masses are naturally explained. Furthermore, $\text{SU}(5) \times \text{SU}(5)$ theories can be easily embedded in superstring constructions [17, 18] which aim at unifying gravity with electroweak and strong forces. In what concerns unification, it is worth noticing that the same discrete permutation symmetry that guarantees the left-right nature of $\text{SU}(5) \times \text{SU}(5)$ (i.e. the one-to-one correspondence among left and right matter field representations) also leads to the unification of gauge couplings into a single constant.

If one assumes that the $\text{SU}(5) \times \text{SU}(5)$ group breaks directly to the SM gauge group $\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$ at the unification scale Λ , then the three SM gauge couplings g_a ($a = s, w, y$) meet together into a single value,

$$\alpha_U = k_3 \alpha_s = k_2 \alpha_w = k_1 \alpha_y, \quad (1.1)$$

where $\alpha_a = g_a^2/(4\pi)$. The coefficients k_i are group factors, $k_i = (\text{Tr } T_i^2)/(\text{Tr } T^2)$, ($i = 1, 2, 3$), where T and T_i are generators of the GUT group properly normalized over the full group and its SM subgroup G_i , respectively. For $\text{SU}(5) \times \text{SU}(5)$ one obtains the non-canonical values $k_1 = 13/3$, $k_2 = 1$ and $k_3 = 2$. The corresponding weak mixing angle at the unification scale is given by

$$\sin^2 \theta_W = \frac{\alpha_y}{\alpha_y + \alpha_w} = \frac{1}{1 + k_1/k_2} = \frac{3}{16}. \quad (1.2)$$

It is commonly believed that this value cannot be reconciled with measurements at the electroweak scale, since it is rather small and, in general, $\sin^2 \theta_W$ decreases from high to low energies [17–19]. Yet, if some appropriate representations are taken into account in the renormalization group evolution of the gauge couplings, this may not be the case. In particular, we shall show that the inclusion of the $(\overline{15}, 1) + (1, 15)$ and their conjugate $(15, 1) + (1, \overline{15})$ representations is sufficient to drive $\sin^2 \theta_W$ to the correct value. This is due to the fact that the $\text{SU}(2)_L$ triplets contained in the 15 and $\overline{15}$ representation of $\text{SU}(5)_L$ strongly adjust the α_w coupling constant. It is also remarkable that the above representations play a crucial role in implementing the seesaw mechanism for neutrino masses.

In this work we revive the idea of grand unification in the supersymmetric version of the left-right $\text{SU}(5) \times \text{SU}(5)$ gauge group. Our aim is to demonstrate that, with the addition of a minimal particle content, it is possible not only to unify the SM gauge coupling constants into a single GUT value, but also to bring the theory into agreement with the electroweak observational data. The paper is organized as follows. In Sec. 2 we introduce the particle content of the model and discuss possible

breaking patterns to the SM gauge group. We also briefly address the question of fermion masses in the context of the generalized seesaw. The unification of gauge couplings at one-loop and two-loop levels is studied in Sec. 3 and a general numerical analysis is presented in Sec. 4. Finally, our concluding remarks are given in Sec. 5.

2 The model

The supersymmetric left-right $\text{SU}(5) \times \text{SU}(5)$ gauge group contains two copies per generation of the usual SUSY $\text{SU}(5)$ theory. In the left-handed picture, the $(\bar{5}+10, 1)$ fermion representations, denoted by ψ and χ , are given by

$$\psi = \begin{bmatrix} D_1^c \\ D_2^c \\ D_3^c \\ e \\ -\nu \end{bmatrix} \sim (\bar{5}, 1), \quad \chi = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & U_3^c & -U_2^c & -u_1 & -d_1 \\ -U_3^c & 0 & U_1^c & -u_2 & -d_2 \\ U_2^c & -U_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -E^c \\ d_1 & d_2 & d_3 & E^c & 0 \end{bmatrix} \sim (10, 1), \quad (2.1)$$

while the $(1, 5 + \bar{10})$ fields, represented by ψ^c and χ^c , are

$$\psi^c = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ e^c \\ -\nu^c \end{bmatrix} \sim (1, 5), \quad \chi^c = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & U_3 & -U_2 & -u_1^c & -d_1^c \\ -U_3 & 0 & U_1 & -u_2^c & -d_2^c \\ U_2 & -U_1 & 0 & -u_3^c & -d_3^c \\ u_1^c & u_2^c & u_3^c & 0 & -E \\ d_1^c & d_2^c & d_3^c & E & 0 \end{bmatrix} \sim (1, \bar{10}). \quad (2.2)$$

The multiplets of Eqs. (2.1) and (2.2) have extra fermions beyond those present in the SM: the vector-like fermions (U, U^c, D, D^c, E, E^c) and the well-motivated right-handed neutrino, ν^c . There is no vector-like analog of the neutrino.

To discuss the breaking scheme to the SM gauge group, one needs to specify the Higgs content. Among the different possibilities, here we consider the following pattern:

$$\begin{aligned} & \text{SU}(5)_L \times \text{SU}(5)_R \\ & \quad \downarrow \Lambda \\ & \text{SU}(3)_L \times \text{SU}(2)_L \times \text{U}(1)_L \times \text{SU}(3)_R \times \text{SU}(2)_R \times \text{U}(1)_R \\ & \quad \downarrow \Lambda_{LR} \\ & \text{SU}(3)_C \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} \\ & \quad \downarrow v_R \\ & \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \\ & \quad \downarrow v_L \\ & \text{SU}(3)_C \times \text{U}(1)_{em}. \end{aligned} \quad (2.3)$$

We identify $\text{SU}(3)_C$ with the $\text{SU}(3)_{L+R}$ diagonal subgroup and $\text{U}(1)_{B-L}$ with $\text{U}(1)_{L+R}$. The breaking energy scales Λ , Λ_{LR} and v_R are determined by the Higgs content of the

model. In this implementation, we need the adjoint representations of both $SU(5)$ subgroups. We introduce $\Phi_L \sim (24, 1)$ and $\Phi_R \sim (1, 24)$, which accomplish the first breaking of $SU(5)_L \times SU(5)_R$ at the scale Λ but preserve the discrete left-right symmetry. To achieve the left-right symmetry breaking at the scale Λ_{LR} , the Higgs fields $\omega \sim (5, \bar{5})$, $\bar{\omega} \sim (\bar{5}, 5)$, $\Omega \sim (10, \bar{10})$ and $\bar{\Omega} \sim (\bar{10}, 10)$ are introduced¹. The last two steps in the pattern (2.3) are driven by the additional Higgs fields $\phi_R \sim (1, \bar{5})$, $\phi_R^c \sim (1, 5)$ and $\phi_L \sim (5, 1)$, $\phi_L^c \sim (\bar{5}, 1)$, respectively. Finally, as mentioned in the Introduction, the representations $T_L \sim (15, 1)$, $T_L^c \sim (\bar{15}, 1)$, $T_R \sim (1, \bar{15})$ and $T_R^c \sim (1, 15)$ turn out to be crucial for unification and are responsible for the Majorana masses of neutrinos.

One of the attractive features of the $SU(5) \times SU(5)$ theory is the possibility of a generalized seesaw mechanism to give masses to all SM fermions through the heavy vector-like fermions [19]. The Yukawa contribution to the superpotential reads as

$$W_Y = \psi^c Y_1 \omega \psi + \chi^c Y_2 \Omega \chi + \sqrt{2} \psi Y_3 \chi \phi_L^c + \sqrt{2} \psi^c Y_3 \chi^c \phi_R^c + \frac{1}{4} \chi Y_4 \chi \phi_L + \frac{1}{4} \chi^c Y_4 \chi^c \phi_R, \quad (2.4)$$

where Y_i denote the Yukawa coupling matrices. We choose the breaking directions as $\langle \omega \rangle_k^k = \langle \Omega \rangle_{12}^{12} = \langle \Omega \rangle_{23}^{23} = \langle \Omega \rangle_{31}^{31} = \langle \Omega \rangle_{45}^{45} = \Lambda_{LR}$, $k = 1, 2, 3$ and $\langle \phi_{L,R} \rangle = (0, 0, 0, 0, v_{uL,R})^T$, $\langle \phi_{L,R}^c \rangle = (0, 0, 0, 0, v_{dL,R})^T$, with $v_{L,R}^2 = v_{uL,R}^2 + v_{dL,R}^2$. The final mass contribution to all charged fermions can then be written as

$$-\mathcal{L}_m = (u \ U) \begin{pmatrix} 0 & Y_4 v_{uL} \\ Y_4 v_{uR} & -Y_2 \Lambda_{LR} \end{pmatrix} \begin{pmatrix} u^c \\ U^c \end{pmatrix} + (d \ D) \begin{pmatrix} 0 & Y_3 v_{dL} \\ Y_3^T v_{dR} & -Y_1 \Lambda_{LR} \end{pmatrix} \begin{pmatrix} d^c \\ D^c \end{pmatrix} + \\ (e \ E) \begin{pmatrix} 0 & Y_3^T v_{dL} \\ Y_3 v_{dR} & -Y_2 \Lambda_{LR} \end{pmatrix} \begin{pmatrix} e^c \\ E^c \end{pmatrix}. \quad (2.5)$$

By means of the above procedure a generalized type-I seesaw mechanism can be implemented for all light quarks and charged leptons, provided that the vector-like fermion masses, which are proportional to the Λ_{LR} scale, are heavy enough and $v_L v_R \ll \Lambda_{LR}^2$. As it turns out, heavy vector-like fermion masses are also required for a successful unification of gauge couplings. For the sake of simplicity, we shall assume that the breaking pattern (2.3) to the SM gauge group occurs at a unique energy scale, *i.e.* $v_R \approx \Lambda_{LR} \approx \Lambda$. In this case the fermion mass spectrum has the approximate seesaw form $m_f = \mathcal{O}(y_f v_L)$ and $M_V = \mathcal{O}(y_V \Lambda_{LR})$, for light and heavy fermions, respectively. The precise realization of this generalized seesaw for fermions is beyond the scope of this work. It is our aim, instead, to discuss in detail how gauge couplings unify in this theory.

¹Alternatively, one could break directly the left-right symmetry at the scale $\Lambda_{LR} = \Lambda$ without the need of the adjoint Higgs fields in the $(24, 1)$ and $(1, 24)$ representations.

For the neutrino sector, the relevant terms in the superpotential are

$$W_N = \sqrt{2} Y_5 (\psi \psi T_L + \psi^c \psi^c T_R), \quad (2.6)$$

if one assumes R-parity conservation. Then, introducing two additional supermultiplets, $(5, \bar{5})$ and $(\bar{5}, 5)$, with vacuum alignment in the lepton doublet direction, light neutrinos would acquire masses through the conventional (type-I and/or type-II) seesaw mechanisms. It is worth noticing that, in the absence of the Higgs multiplets ϕ_R , ϕ_R^c , ϕ_L and ϕ_L^c , R-parity is automatically conserved² [18]. In the latter case, quark and charged lepton masses would arise from higher dimension operators instead of the generalized seesaw Lagrangian terms given in Eq. (2.5).

3 Gauge coupling unification

The two-loop renormalization group equations (RGE) for the gauge coupling constants α_i ($i = 1, 2, 3$) can be written in the form

$$\frac{d}{dt} \alpha_i^{-1} = -\frac{b_i}{2\pi k_i} - \frac{1}{8\pi^2} \sum_j \frac{b_{ij} \alpha_j}{k_i k_j} - \frac{1}{32\pi^3 k_i} \sum_{f=u,d,e} C_{if} \text{Tr} (Y_f^\dagger Y_f), \quad (3.1)$$

where $\alpha_1 = k_1 \alpha_y$, $\alpha_2 = k_2 \alpha_w$ and $\alpha_3 = k_3 \alpha_s$; b_i are the usual one-loop beta coefficients; b_{ij} and C_{if} are the two-loop beta coefficients (see Appendix A). The quantities Y_f denote the quark and lepton Yukawa coupling matrices. At the unification scale Λ , the gauge couplings α_i obey the relation $\alpha_U = \alpha_1 = \alpha_2 = \alpha_3$ (cf. Eq. (1.1)).

To get some insight into the unification in the one-loop approximation, let us define the effective beta coefficients B_i [20],

$$B_i \equiv \frac{1}{k_i} \left(b_i + \sum_I b_i^I r_I \right), \quad (3.2)$$

where

$$r_I = \frac{\ln(\Lambda/M_I)}{\ln(\Lambda/M_Z)}. \quad (3.3)$$

In the above expression, M_I denotes an intermediate energy scale between the electroweak scale M_Z and the GUT scale Λ , and the coefficients b_i^I account for the new contribution to the one-loop beta functions b_i above the threshold M_I . It is also convenient to introduce the differences $B_{ij} \equiv B_i - B_j$, such that

$$B_{ij} = B_{ij}^{\text{SM}} + \sum_I \Delta_{ij}^I r_I, \quad (3.4)$$

where B_{ij}^{SM} corresponds to the SM particle contribution and

$$\Delta_{ij}^I = \frac{b_i^I}{k_i} - \frac{b_j^I}{k_j}. \quad (3.5)$$

²Terms in the superpotential such as $\psi \phi_L$, $\chi \phi_L^c \phi_L^c$, $T_L \psi \phi_L^c$ and $T_L \chi \phi_L$ violate R-parity.

The following B -test is then obtained,

$$B \equiv \frac{B_{23}}{B_{12}} = \frac{\sin^2 \theta_W - \frac{k_2}{k_3} \frac{\alpha}{\alpha_s}}{\frac{k_2}{k_1} - \left(1 + \frac{k_2}{k_1}\right) \sin^2 \theta_W}, \quad (3.6)$$

together with the GUT scale relation

$$B_{12} \ln \left(\frac{\Lambda}{M_Z} \right) = \frac{2\pi}{\alpha} \left[\frac{1}{k_1} - \left(\frac{1}{k_1} + \frac{1}{k_2} \right) \sin^2 \theta_W \right]. \quad (3.7)$$

Notice that the right-hand sides of Eqs. (3.6) and (3.7) depend only on low-energy electroweak data and the group factors k_i . Adopting the following experimental values at M_Z [21]

$$\alpha^{-1} = 127.916 \pm 0.015, \quad (3.8)$$

$$\sin^2 \theta_W = 0.23116 \pm 0.00013, \quad (3.9)$$

$$\alpha_s = 0.1184 \pm 0.0007, \quad (3.10)$$

the above relations read as

$$\begin{aligned} B &= 0.718 \pm 0.003, \\ B_{12} \ln \left(\frac{\Lambda}{M_Z} \right) &= 185.0 \pm 0.2, \end{aligned} \quad (3.11)$$

in the canonical GUT models with $k_i = (5/3, 1, 1)$, *e.g.* in $\text{SU}(5)$ and $\text{SO}(10)$. On the other hand, for the $\text{SU}(5) \times \text{SU}(5)$ model where $k_i = (13/3, 1, 2)$ one obtains

$$\begin{aligned} B &= -3.687 \pm 0.012, \\ B_{12} \ln \left(\frac{\Lambda}{M_Z} \right) &= -43.19 \pm 0.13. \end{aligned} \quad (3.12)$$

The coefficients B_{ij} that appear in the left-hand sides of Eqs. (3.6) and (3.7) strongly depend on the particle content of the theory. For instance, considering the SM particles with n_H light Higgs doublets, one has $b_1 = 20/3 + n_H/6$, $b_2 = -10/3 + n_H/6$ and $b_3 = -7$, so that these coefficients are given by

$$B_{12} = \frac{22}{3} - \frac{n_H}{15}, \quad B_{23} = \frac{11}{3} + \frac{n_H}{6}. \quad (3.13)$$

In the supersymmetric case they become

$$B_{12} = \frac{22}{3} - \frac{n_H}{15} - \left(\frac{4}{3} + \frac{2n_H}{15} \right) r_S, \quad B_{23} = \frac{11}{3} + \frac{n_H}{6} + \left(-\frac{2}{3} + \frac{n_H}{3} \right) r_S, \quad (3.14)$$

with the “running weight” $r_S \simeq 0.93$, for a low SUSY threshold $M_S \simeq 1$ TeV and a unification scale $\Lambda \simeq 10^{16}$ GeV.

It is interesting to notice that Eqs. (3.13) and (3.14) together with the constraint (3.11) allow to determine the number of the light Higgs doublets that would be required for the unification in the canonical GUT models,

$$n_H = 110 \left(\frac{2B-1}{2B+5} \right) \approx 7 \quad (\text{SM}), \quad (3.15)$$

$$n_H = 10 \left(\frac{11-2r_s}{1+2r_s} \right) \left(\frac{2B-1}{2B+5} \right) \approx 2 \quad (\text{MSSM}). \quad (3.16)$$

Clearly, the B-test fails badly in the SM case which possesses only one Higgs doublet, while Eq. (3.16) just corroborates the fact that the gauge couplings in the MSSM seemingly unify at one-loop level. Would one take only the MSSM particle content into account, the B-test would also fail badly in the SUSY $\text{SU}(5) \times \text{SU}(5)$ case. Indeed, in such a case $B \approx 1.625$ which is far above the required value given in Eq. (3.12) and, hence, the need for extra particles with suitable B_{ij} coefficients. In Table 1 we present the relevant contributions Δ_{ij} to the B_{ij} coefficients of the SUSY $\text{SU}(5) \times \text{SU}(5)$ model which include, besides the MSSM threshold, the triplet Σ_3 and octet Σ_8 belonging to the $(24, 1)$ representation, the triplets T, T^c in the $(15, 1) + (\overline{15}, 1)$ representation as well as the exotic vector-like chiral multiplets U, U^c, D, D^c and E, E^c .

Table 1. The Δ_{ij} contributions to the B_{ij} coefficients in the $\text{SU}(5) \times \text{SU}(5)$ case. The SM contribution to the coefficients are $B_{12}^{\text{SM}} = 185/39$ and $B_{23}^{\text{SM}} = 1/3$.

	MSSM	Σ_3	Σ_8	T	U	D	E
Δ_{12}	-125/39	-2	0	-34/13	24/13	6/13	18/13
Δ_{23}	13/6	2	-3/2	4	-3/2	-3/2	0

Since Eqs. (3.12) require $B_{12} < 0$ and $B_{23} > 0$, it becomes clear from Table 1 that Σ_3 and T improve unification, while U, D and E act in the opposite manner and, therefore, should be heavy enough. For illustration, in Fig. 1 we plot the one-loop running of the gauge couplings for the SUSY $\text{SU}(5)$ and $\text{SU}(5) \times \text{SU}(5)$ theories, assuming a common unification scale, $\Lambda = 2 \times 10^{16}$ GeV. The SUSY threshold M_S is chosen in both cases at 1 TeV. For the $\text{SU}(5) \times \text{SU}(5)$ case, we assume a common mass scale M_Σ for Σ_3 and Σ_8 , and for the vector-like particles U, D, E we set their mass scale $M_V = \Lambda$. The one-loop unification then demands $M_\Sigma \simeq 10$ TeV and the triplets T, T^c to have a mass $M_T \simeq 10^9$ GeV. The evolution of $\sin^2 \theta_W$ at one-loop level is given in Fig. 2. As anticipated in the Introduction, adding the appropriate $\text{SU}(5) \times \text{SU}(5)$ representations is essential for driving the running of $\sin^2 \theta_W$ from the low value 3/16 at GUT scale to its correct value at the electroweak scale.

One may wonder whether two-loop effects significantly modify the above picture. The example presented in Fig. 3 shows that, although the values of the gauge

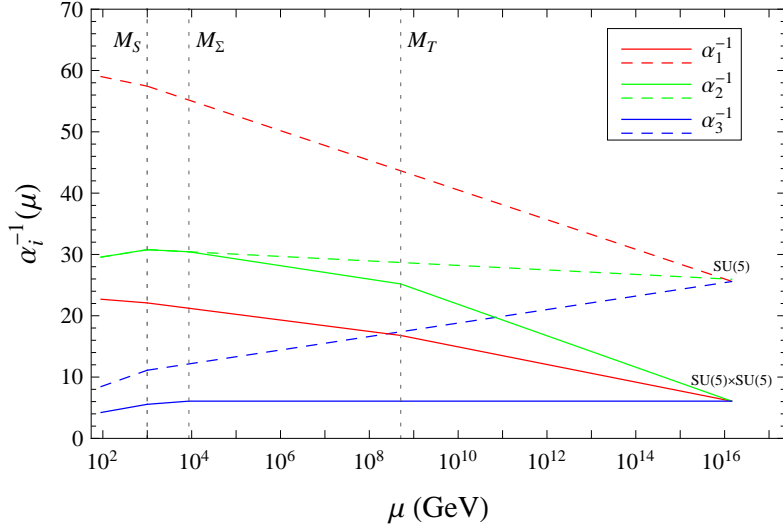


Figure 1. The gauge coupling running at one-loop level for the canonical SU(5) MSSM (dashed lines) and the SU(5) \times SU(5) theory (solid lines), assuming the same unification scale, $\Lambda \simeq 2 \times 10^{16}$ GeV. The SUSY scale is fixed at $M_S = 1$ TeV. Notice that for the non-canonical case one needs Σ_3 and Σ_8 close to $M_\Sigma = 10$ TeV and the triplets T, T^c at a higher scale near $M_T = 10^9$ GeV.

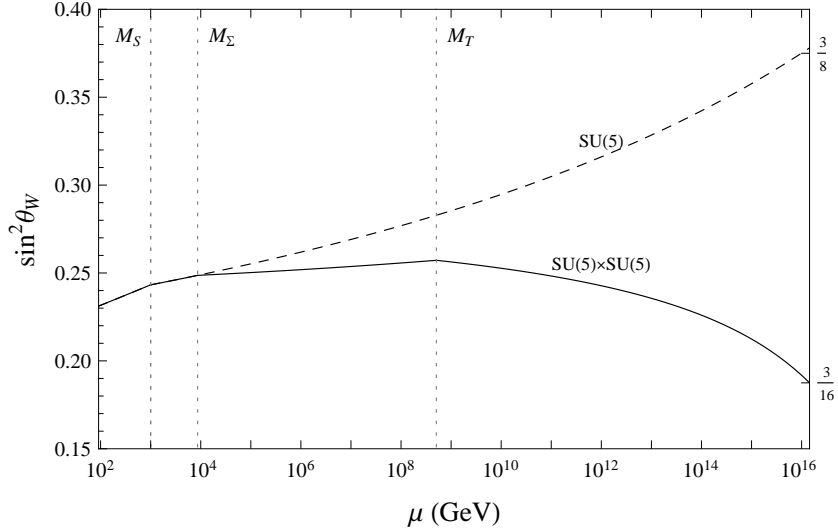


Figure 2. The evolution of $\sin^2 \theta_W$ at one-loop level for the canonical SU(5) MSSM (dashed line) and the SU(5) \times SU(5) theory (solid line), assuming $\Lambda \simeq 2 \times 10^{16}$ GeV, $M_S = 1$ TeV, $M_\Sigma = 10$ TeV and $M_T = 10^9$ GeV.

couplings as a function of the energy scale μ are essentially unchanged, the two-loop effects tend to increase both M_Σ and M_T scales. In the next section we shall perform a two-loop numerical analysis in order to determine the full range of the relevant

intermediate mass scales.

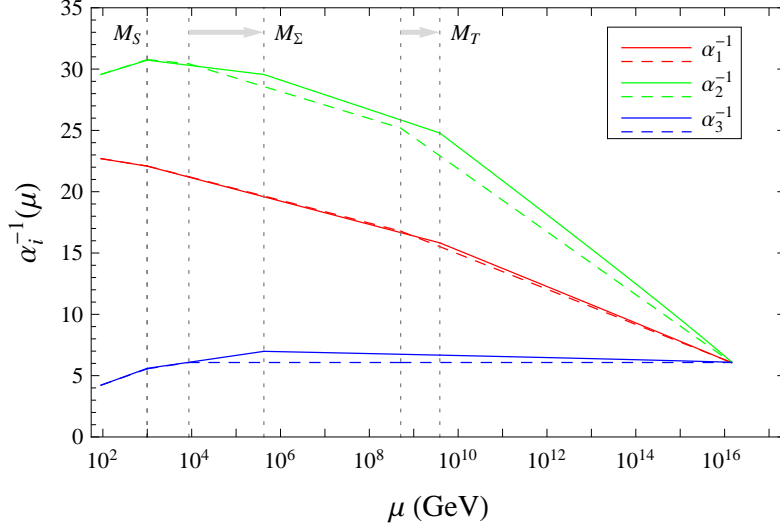


Figure 3. Comparison of the $SU(5) \times SU(5)$ running of gauge couplings at one-loop level (dashed lines) and two-loop level (solid lines). For a fixed $M_S = 1$ TeV and the same unification scale $\Lambda \simeq 2 \times 10^{16}$ GeV, two-loop effects increase the intermediate scales M_Σ and M_T .

4 Numerical analysis

In this section we present a general numerical analysis of the two-loop gauge coupling unification of the $SU(5) \times SU(5)$ model sketched in Sec. 2. We adopt the \overline{DR} scheme, which is appropriate for the two-loop renormalization group evolution in supersymmetric models. The measure of unification used here is given by the quantity

$$\epsilon = \sqrt{(\alpha_{1\Lambda}^{-1} - \alpha_{2\Lambda}^{-1})^2 + (\alpha_{1\Lambda}^{-1} - \alpha_{3\Lambda}^{-1})^2 + (\alpha_{2\Lambda}^{-1} - \alpha_{3\Lambda}^{-1})^2}, \quad (4.1)$$

which measures the “distance” between the couplings $\alpha_{i\Lambda}^{-1} \equiv \alpha_i^{-1}(\Lambda)$ at the unification scale Λ . Alternatively, one could use the quantity [22]

$$R = \frac{\max(\alpha_{1\Lambda}, \alpha_{2\Lambda}, \alpha_{3\Lambda})}{\min(\alpha_{1\Lambda}, \alpha_{2\Lambda}, \alpha_{3\Lambda})}, \quad (4.2)$$

which measures the amount of non-unification between the largest and the smallest gauge coupling value at the scale Λ . We have verified that both quantities lead to similar unification constraints. In particular, requiring $\epsilon \lesssim 0.1$ would correspond to $R - 1 \lesssim 0.07$.

Solving for the one-loop RGE of gauge couplings in the MSSM, and assuming a SUSY threshold $M_S = 1$ TeV, the measure ϵ attains its minimum value, $\epsilon \simeq 0.50$,

for $\Lambda \simeq 1.44 \times 10^{16}$ GeV. On the other hand, at two-loop level, its minimum is $\epsilon \simeq 0.18$ for $\Lambda \simeq 1.38 \times 10^{16}$ GeV, so that two-loop effects significantly improve unification. Inspired by the MSSM results, in our study we choose values of $\epsilon \leq 0.1$ as the criterion for unification. One can then expect that threshold effects would be sufficient to yield a perfect unification.

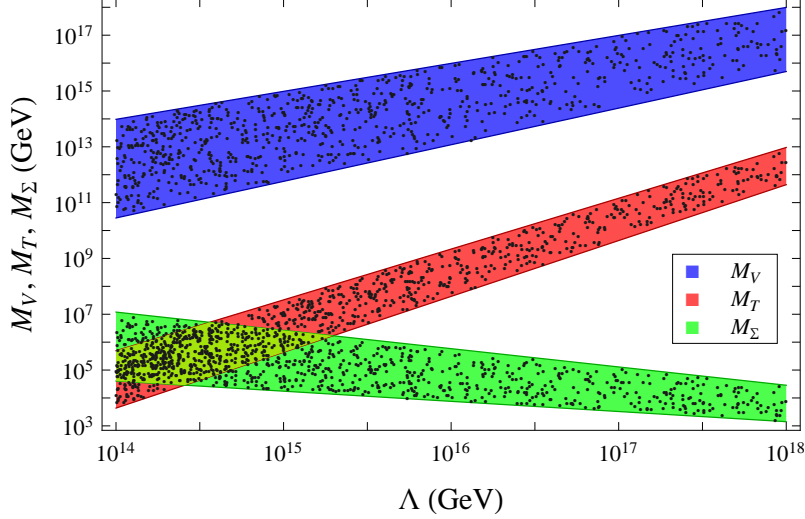


Figure 4. Intermediate mass scales M_V , M_T and M_Σ as functions of the unification scale Λ in the $SU(5) \times SU(5)$ model. The delimited color regions correspond to solutions $\alpha_{i\Lambda}^{-1}$ ($i = 1, 2, 3$) with a unification measure $\epsilon \leq 0.1$ at two loops.

We proceed to integrate numerically the two-loop RGEs in Eqs. (3.1) from the electroweak scale M_Z to a randomly chosen unification scale $\Lambda \gtrsim 10^{14}$ GeV. The intermediate vector-like fermion mass scale M_V , and that of the triplet scalar, M_T , as well as the common scale M_Σ for Σ_3 and Σ_8 , are also randomly taken. The SUSY threshold scale is fixed at $M_S = 1$ TeV. At two-loop level, the parameter space for the three relevant quantities, M_V , M_T , and M_Σ , is given as a function of the unification scale Λ in Fig. 4. We notice that every point corresponds to a different solution which has passed the criterion $\epsilon \leq 0.1$. As can be easily seen from the figure, the triplet mass scale M_T can be close to the SUSY breaking mass scale M_S for a low unification scale $\Lambda \simeq 10^{14}$ GeV. As Λ increases, the value of M_T also increases. We find $4.5 \times 10^3 \text{ GeV} \lesssim M_T \lesssim 1.2 \times 10^{13} \text{ GeV}$ for $10^{14} \text{ GeV} \lesssim \Lambda \lesssim 10^{18} \text{ GeV}$. In contrast, the common mass scale M_Σ decreases smoothly as Λ increases and, for $\Lambda \sim 10^{18}$ GeV, can be as low as 1 TeV. The allowed mass range is $1.2 \times 10^3 \text{ GeV} \lesssim M_\Sigma \lesssim 2.7 \times 10^7 \text{ GeV}$. We also note that, when $\Lambda \simeq 10^{14-15}$ GeV, both mass scales, M_T and M_Σ , can be of the same order of magnitude. When compared to other intermediate states, vector-like fermions require a much higher mass scale. For $\Lambda \simeq 10^{14}$ GeV, we find the lower bound $M_V \gtrsim 3.2 \times 10^{10}$ GeV, while

for $\Lambda \simeq 10^{17}$ GeV this bound gets more restrictive, $M_V \gtrsim 10^{15}$ GeV.

We have also verified how sensitive the results are with respect to the variation of the SUSY breaking mass scale. In fact, no significant changes occur and the variation of the SUSY mass scale in the interval $M_S = 1 - 100$ TeV leads only to a slight dispersion of M_Σ towards lower values. No relevant modification is either observed for the parameters in Fig. 4, if one considers a splitting between the masses of the triplet Σ_3 and octet Σ_8 . Motivated by the rich scalar structure, we have also looked for solutions when two additional Higgs doublets are randomly inserted at some new threshold, M_H . The effects of the latter on the mass scales M_V , M_T and M_Σ , given as a function of the unification scale Λ , are shown in Fig. 5. While the inclusion of the two additional Higgs doublets does not significantly affect the parameter region of M_V and M_Σ , it is clear from Fig. 5 that the triplet mass scale M_T is shifted to much higher values, bringing M_T close to the vector-like fermion mass scale for $\Lambda \gtrsim 10^{16}$ GeV. The allowed ranges for the relevant scales are now given by 1.4×10^{10} GeV $\lesssim M_V \lesssim 9.7 \times 10^{17}$ GeV, 9.5×10^5 GeV $\lesssim M_T \lesssim 4.4 \times 10^{16}$ GeV, and 3.9×10^3 GeV $\lesssim M_\Sigma \lesssim 1.4 \times 10^8$ GeV, with M_H varying in the interval 10^3 GeV $\lesssim M_H \lesssim 9.4 \times 10^{17}$ GeV.

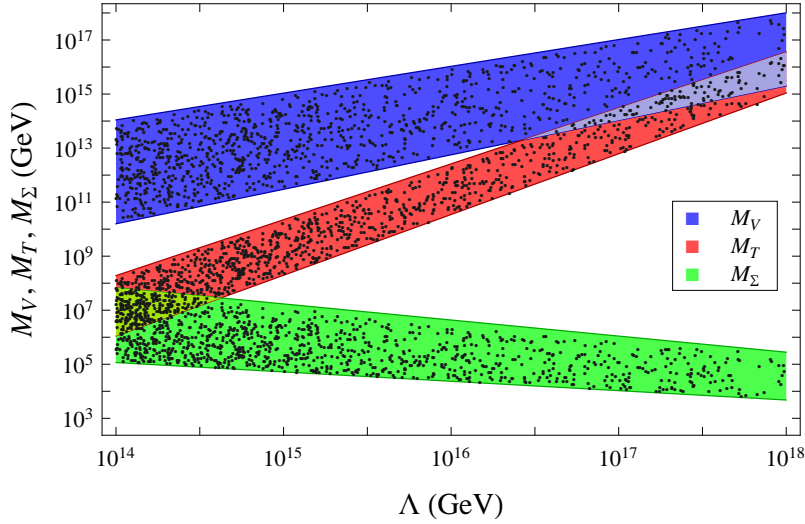


Figure 5. As in Fig. 4, but for the $SU(5) \times SU(5)$ model with two additional Higgs doublets at an intermediate scale. Notice that the triplet mass scale M_T is significantly larger, and reaches values of the order of the exotic fermion mass scale M_V at a high unification scale.

From the above results it becomes clear that the unification scale Λ can reach and even exceed the perturbative string scale, $\Lambda_s \simeq 5.27 \times 10^{17}$ GeV [23, 24]. It is well known that $SU(5) \times SU(5)$ theories can be embedded in the heterotic string context [25–30]. Furthermore, in a minimal string-scale unification setup with vector-like fermions, it is conceivable to have unification of gauge couplings and gravity

at the weakly coupled heterotic string scale [31]. We may ask ourselves whether it is possible to achieve such a unification in the $\text{SU}(5) \times \text{SU}(5)$ framework under consideration. In the heterotic string scenario, an additional constraint on the gauge couplings must be verified at the string scale Λ_s ,

$$\alpha_U = \alpha_{\text{string}} = \frac{1}{4\pi} \left(\frac{\Lambda}{\Lambda_s} \right)^2. \quad (4.3)$$

Requiring $\Lambda \leq \Lambda_s$ in order to be in the perturbative regime, the constraint in Eq. (4.3) clearly implies a lower bound on the unified gauge coupling, namely, $\alpha_U^{-1} \geq 4\pi$. In Fig. 6, we present the upper values of $\alpha_U^{-1} \simeq \alpha_{2\Lambda}^{-1}$ as a function of the unification scale Λ , together with the corresponding values of $\alpha_{\text{string}}^{-1}$. We conclude that string unification cannot be achieved, since α_U^{-1} is very small compared to the required value of $\alpha_{\text{string}}^{-1}$. This conclusion also remains valid when two additional Higgs doublets are included at an intermediate energy scale.

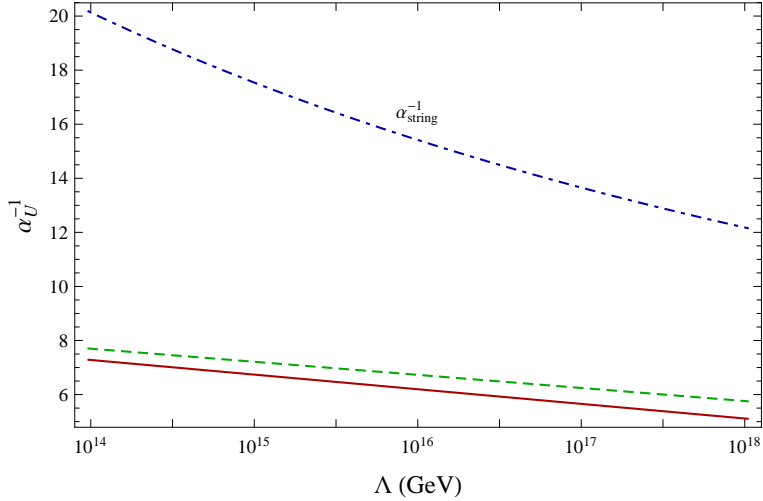


Figure 6. Upper values of α_U^{-1} at two-loop level in the $\text{SU}(5) \times \text{SU}(5)$ model. The solid line corresponds to the $\text{SU}(5) \times \text{SU}(5)$ model, assuming only two light Higgs doublets, while the dashed line corresponds to the case where two extra Higgs doublets are introduced at some intermediate scale.

5 Conclusions

We have investigated the possibility to achieve unification of the SM gauge couplings in the context of a SUSY $\text{SU}(5)_L \times \text{SU}(5)_R$ GUT. For a successful gauge coupling unification, the inclusion of $(\overline{15}, 1) + (1, 15)$ and their conjugates $(15, 1) + (1, \overline{15})$ at an intermediate scale M_T was essential to drive $\sin^2 \theta_W$ to the correct value at the electroweak scale. From the two-loop numerical analysis, we have found that the

intermediate mass scales M_T , M_Σ for Σ_3 , Σ_8 and M_V for the vector-like fermions must be properly chosen to guarantee unification at the required level. As it can be clearly seen from Figs. 4 and 5, there is a wide region allowed for these mass scales.

Models based on $SU(5)_L \times SU(5)_R$ unification enclose many attractive features. Compared with the standard $SU(5)$ GUT, proton decay via dimension-six operators through heavy lepto-quark gauge bosons is suppressed, since at tree level the latter do not mediate transitions involving only light fermions. On the other hand, the presence of the color Higgs triplets H_C^L and H_C^R , contained in the chiral super-quintets ϕ_L , ϕ_L^c , ϕ_R and ϕ_R^c , may induce proton decay through dimension-five operators. Indeed, proton decay arises in the lowest order from the operators $\chi\chi\chi\psi$ and $\chi^c\chi^c\chi^c\psi^c$, which lead to the effective operators $QQQL$ with coefficients proportional to Y_3Y_4/M_{H_C} , for both left and right light matter fields. This requires that the mass scales of left and right color Higgs triplets should be heavy enough, thus constraining the unification scale [11]. In the absence of the fields $\phi_{L,R}$ and $\phi_{L,R}^c$ not only proton is stable at the renormalizable level, but also R-parity is automatically conserved [18]. R-parity invariance is an appealing feature in SUSY theories, since the lightest supersymmetric particle is absolutely stable, thus providing a natural cold dark matter candidate.

Finally, we have shown that, in the minimal $SU(5)_L \times SU(5)_R$ setup considered, it is not possible to achieve the unification of the gauge couplings with the gravitational coupling at the perturbative heterotic string scale. It would be interesting to investigate whether the inclusion of additional representations could help in bringing into agreement the four couplings.

Acknowledgements

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A One-loop and two-loop beta coefficients

In this appendix we collect the β -function coefficients for the relevant particle content of the $SU(5) \times SU(5)$ theory.

Below the SUSY threshold M_S , the β -function coefficients are those of the SM:

$$b_i = (41/6 \ -19/6 \ -7) \ , \quad b_{ij} = \begin{pmatrix} 199/18 & 9/2 & 44/3 \\ 3/2 & 35/6 & 12 \\ 11/6 & 9/2 & -26 \end{pmatrix} . \quad (\text{A.1})$$

Above M_S , the coefficients are the usual MSSM ones:

$$b_i = (11 \ 1 \ -3) \ , \quad b_{ij} = \begin{pmatrix} 199/9 & 9 & 88/3 \\ 3 & 25 & 24 \\ 11/3 & 9 & 14 \end{pmatrix} . \quad (\text{A.2})$$

The two-loop coefficients C_{if} that account for the Yukawa contributions are

$$C_{if} = \begin{pmatrix} 26/3 & 14/3 & 6 \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{pmatrix} . \quad (\text{A.3})$$

We have also the following coefficients for the triplet Σ_3 , the octet Σ_8 , the triplet T , and the vector-like fermions U , D and E :

$$b_i^{\Sigma_3} = (0 \ 2 \ 0) \ , \quad b_{ij}^{\Sigma_3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad (\text{A.4})$$

$$b_i^{\Sigma_8} = (0 \ 0 \ 3) \ , \quad b_{ij}^{\Sigma_8} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 54 \end{pmatrix} , \quad (\text{A.5})$$

$$b_i^T = (6 \ 4 \ 0) \ , \quad b_{ij}^T = \begin{pmatrix} 24 & 48 & 0 \\ 16 & 48 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad (\text{A.6})$$

$$b_i^U = (8 \ 0 \ 3) \ , \quad b_{ij}^U = \begin{pmatrix} 128/9 & 0 & 128/3 \\ 0 & 0 & 0 \\ 16/3 & 0 & 34 \end{pmatrix} , \quad (\text{A.7})$$

$$b_i^D = (2 \ 0 \ 3) \ , \quad b_{ij}^D = \begin{pmatrix} 8/9 & 0 & 32/3 \\ 0 & 0 & 0 \\ 4/3 & 0 & 34 \end{pmatrix} , \quad (\text{A.8})$$

$$b_i^E = (6 \ 0 \ 0) \ , \quad b_{ij}^E = \begin{pmatrix} 24 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad (\text{A.9})$$

which are introduced at the appropriate intermediate scales.

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